TOWARDS A MODEL INDEPENDENT DETERMINATION OF FRAGMENTATION FUNCTIONS[1]

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Abstract

We show that the difference cross sections in unpolarized SIDIS $e+N \to e+h+X$ and pp hadron production $p + p \to h + X$ determine, uniquely and in any order in QCD, the two FFs: $D_u^{h-\bar{h}}$ and $D_d^{h-\bar{h}}$, $h = \pi^\pm, K^\pm$. If both $K^\pm$ and K_s^0 are measured, then $e^+e^- \to K + X$, $e + N \to e + K + X$ and $p + p \to K + X$ yield independent measurements of $(D_u - D_d)^{K^+ + K^-}$. In a combined fit to $K^\pm$ and K_s^0 production data from e^+e^- collisions, $(D_u - D_d)^{K^+ + K^-}$ is obtained and compared to conventional parametrizations.

1 Introduction

Now, that the new generation of high energy scattering experiments with a final hadron h detected are taking place, it has become clear that in order to obtain the correct information about quark-lepton interactions, not only knowledge of the parton distribution functions (PDFs) are important, but a good knowledge of the fragmentation functions (FFs) D_i^h, that determine the transition of partons i into hadrons h, are equally important. The PDFs and the FFs are the two basic ingredients that have to be correctly extracted from experiment. Here we discuss the FFs.

The most direct way to determine the FFs is the total cross section for one-particle inclusive production in e^+e^- annihilation:

$$e^+e^- \to h + X, \qquad h = \pi^\pm, K^\pm, p/\bar{p}... \tag{1}$$

However, these processes can determine, in principle, only the combinations

$$D_{u+\bar{u}}^h, \quad D_{d+\bar{d}}^h, \quad D_{s+\bar{s}}^h, \quad D_g^{h+\bar{h}} \quad (D_{q+\bar{q}}^h = D_q^{h+\bar{h}} \equiv D_q^h + D_{\bar{q}}^h), \tag{2}$$

i.e. they cannot distinguish the quark and anti-quark FFs. In order to achieve separate *phenomenological* determination of D_q^h and $D_{\bar{q}}^h$, the one-hadron semi-inclusive processes play an essential role:

$$l + N \to l + h + X \quad \text{and} \quad p + p \to h + X. \tag{3}$$

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The factorization theorem implies that the FFs are universal, i.e. in (1) and (3) the FFs are the same. However, in (3) the hadron structure enters and when analyzing the data, usually different theoretical assumptions have to be made.

Schematically the cross sections for (1) and (3) can be written in the form:

$$d\sigma^h_{e^+e^-}(z) \simeq \sum_c \hat{\sigma}^c_{e^+e^-} \otimes D^{h+\bar{h}}_c \tag{4}$$

$$d\sigma^h_N(x,z) \simeq \sum_{q,c} f_q(x) \otimes \hat{\sigma}^c_{lq} \otimes D^h_c \tag{5}$$

$$d\sigma^h_{pp}(x,z) \simeq \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}^c_{ab} \otimes D^h_c \tag{6}$$

where $\hat{\sigma}^c$ are the corresponding, perturbatively QCD calculable, parton-level cross sections for producing a parton c, f_a are the unpolarized PDFs. At present several sets of FFs exist and there is a significant disagreement among some of the FFs.

In this talk we present a model independent approach, developed in [1], which suggests that if instead of $d\sigma^h_N$ and $d\sigma^h_{pp}$ one works with the difference cross sections for producing hadrons and producing their antiparticles, i.e. with data on $d\sigma^{h-\bar{h}}_N \equiv d\sigma^h_N - d\sigma^{\bar{h}}_N$ or $d\sigma^{h-\bar{h}}_{pp} \equiv d\sigma^h_{pp} - d\sigma^{\bar{h}}_{pp}$, one obtains information about the non-singlet (NS) combinations $D^{h-\bar{h}}_q$. This is the complementary to $D^{h+\bar{h}}_q$ quantity, measured in (1), that would allow to determine D_q and $D_{\bar{q}}$ without assumptions.

Further this method is applied to charged and neutral kaon e^+e^--production data to determine directly the non-singlet $(D_u - D_d)^{K^++K^-}$ and compare to conventional global fit analysis.

2 Difference cross sections with $\pi^\pm$ and $K^\pm$

From C-invariance of strong interactions it follows:

$$D^{h^+-h^-}_g = 0, \qquad D^{h^+-h^-}_q = -D^{h^+-h^-}_{\bar{q}} \tag{7}$$

which, applied to (3), eliminates $D^{h^+-h^-}_g$ and $D^{h^+-h^-}_{\bar{q}}$ in the difference cross sections:

$$d\sigma^{h^+-h^-}_N = d\sigma^{h^+}_N - d\sigma^{h^-}_N \quad \text{and} \quad d\sigma^{h^+-h^-}_{pp} = d\sigma^{h^+}_{pp} - d\sigma^{h^-}_{pp} \tag{8}$$

This implies that, in any order of QCD, $d\sigma^{h^+-h^-}_N$ and $d\sigma^{h^+-h^-}_{pp}$ are expressed solely in terms of the NS combinations of the FFs. In NLO we have:

$$d\sigma^{h^+-h^-}_p(x,z,Q^2) = \frac{1}{9}\left[4u_V \otimes D_u + d_V \otimes D_d + s_V \otimes D_s\right]^{h^+-h^-} \otimes (1 + \frac{\alpha_s}{2\pi}C_{qq}) \tag{9}$$

$$d\sigma^{h^+-h^-}_d(x,z,Q^2) = \frac{1}{9}\left[(u_V + d_V) \otimes (4D_u + D_d) + 2s_V \otimes D_s\right]^{h^+-h^-} \otimes (1 + \frac{\alpha_s}{2\pi}C_{qq}) \tag{10}$$

$$E^h \frac{d\sigma^{h^+-h^-}_{pp}}{d^3P^h} = \frac{1}{\pi} \int dx_a \int dx_b \int \frac{dz}{z} \times$$
$$\times \sum_{q=u,d,s} \left[L_q(x_b,t,u)q_V(x_a) + L_q(x_a,u,t)q_V(x_b)\right] D^{h^+-h^-}_q(z) \tag{11}$$

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where u_V and d_V are the valence quarks PDFs, $s_V = s - \bar{s}$ and L_q, given explicitly in [1], are functions of the *known* quark densities $q + \bar{q}$ and partonic cross sections.

Common for the difference cross sections (9) - (11) is that they all have the same structure: 1) only the non-singelts $D_q^{h-\bar{h}}$ enter and 2) they enter in the combination $q_V D_q^{h-\bar{h}}$. This implies that the contributions of D_u^h and D_d^h are enhanced by the large valence quark densities, while D_s^h is suppressed by the small factor $(s - \bar{s})$. Recently a strong bound on $(s - \bar{s})$ was obtained from neutrino experiments $- |s - \bar{s}| \leq 0.025$ [2], which implies that the contribution from D_s^h can be safely neglected. Thus, the *ep*, *ed* and *pp* semi-inclusive difference cross sections provide three independent measurements for the two unknown FFs $D_u^{h^+ - h^-}$ and $D_d^{h^+ - h^-}$. Note that the SIDIS cross sections involve only u_V and d_V, which are the best known parton densities, with 2%-3% accuracy at $x \lesssim 0.7$.

Further information can be obtained specifying the final hadrons.

1) If $h = \pi^\pm$ the difference cross sections will determine, without any assumptions, $D_u^{\pi^+ - \pi^-}$ and $D_d^{\pi^+ - \pi^-}$ which would allow to test the usually made assumption

$$D_u^{\pi^+ - \pi^-} = -D_d^{\pi^+ - \pi^-}. \tag{12}$$

In [3] it was suggested, for the first time, that this relation might be violated up to 10 %.

2) If $h = K^\pm$ the difference cross sections will determine, without any assumptions, $D_u^{K^+ - K^-}$ and $D_d^{K^+ - K^-}$ which would allow to test the usually made assumption

$$D_d^{K^+ - K^-} = 0. \tag{13}$$

One can formulate the above results also like this: If relation $D_u^{\pi^+ - \pi^-} = -D_d^{\pi^+ - \pi^-}$ (or $D_d^{K^+ - K^-} = 0$) holds, then Eqs. (9), (10) and (11) for $h = \pi^\pm$ (or $h = K^\pm$) are expressed solely in terms of $D_u^{\pi^+ - \pi^-}$ (or $D_u^{K^+ - K^-}$) and thus look particularly simple.

3   Difference cross sections with $K^\pm$ and K_s^0

If in addition to the charged $K^\pm$ also neutral kaons $K_s^0 = (K^0 + \bar{K}^0)/\sqrt{2}$ are measured, no new FFs are introduced into the cross-sections. We show that the combination

$$\sigma^{\mathcal{K}} \equiv \sigma^{K^+} + \sigma^{K^-} - 2\sigma^{K_s^0} \tag{14}$$

in the considered three types of semi-inclusive processes (1) and (3):

$$e^+ + e^- \to K + X, \tag{15}$$

$$e + N \to e + K + X, \tag{16}$$

$$p + p \to K + X, \tag{17}$$

$K = K^\pm, K_s^0$, always measures only the NS combination $(D_u - D_d)^{K^+ + K^-}$. This result relies only on SU(2) invariance for the kaons, that relates $D_q^{K_s^0}$ to $D_q^{K^+ + K^-}$ and does not involve *any* assumptions about PDFs or FFs, it holds in any order in QCD. As $(D_u - D_d)^{K^+ + K^-}$, obtained in this way, is model independent it would be interesting to compare it to the existing parametrizations from $e^+ e^-$ data, obtained using various assumptions.

4 $(D_u - D_d)^{K^+ + K^-}$ from $e^+ e^-$ kaon production

The most precisely measured and theoretically calculated process is (15). In NLO we have [1]:

$$d\sigma^{\mathcal{K}}_{e^+e^-}(z, Q^2) = \frac{8\pi \, \alpha_{em}^2}{s} \, (\hat{e}_u^2 - \hat{e}_d^2)(1 + \frac{\alpha_s}{2\pi} \, C_q \otimes) \, (D_u - D_d)^{K^+ + K^-}(z, Q^2). \qquad (18)$$

where $\hat{e}_q^2(s)$ are the quark electro-weak charges. Using (18) we determine [6] $(D_u - D_d)^{K^+ + K^-}$ from the available data on $K^\pm$ and K_s^0 production in $e^+ e^- \to (\gamma, Z) \to K + X$, $K = K^\pm, K_s^0$ and compare it to those obtained in global fit analysis.

Our analysis has several advantages: it allows for the first time to extract $(D_u - D_d)^{K^+ + K^-}$ without any assumptions about the FFs (commonly used in global fit analysis) and without any correlations to other FFs (and especially to $D_g^{K^\pm}$), it allows to use data at much lower values of z than in global fit analysis (being a NS the combination $\sigma^{\mathcal{K}}_{e^+e^-}$ does not contain the unresummed soft gluon logarithms), etc.

We have included the data on $K^\pm$ and K_s^0 production from TASSO, HRS, MARKII, TPC, TOPAZ, ALEPH, DELPHI, OPAL, SLD and CELLO collaborations in the energy intervals $\sqrt{s} = 12 - 14.8, 21.5 - 22, 29 - 35, 42.6 - 44, 58, 91.2$ and $183 - 186$ GeV.

In Fig.1 we present $(D_u - D_d)^{K^+ + K^-}$ in NLO obtained in our approach and from the global fits of the DSS[3], HKNS[4] and AKK08[5] sets. As seen from the Figure, at $z \gtrsim 0.4$ there is an agreement among the different FFs in shape, but our FF is in general larger. The difference becomes more pronounced at $z \lesssim 0.4$. There could be different reasons for this. Most probably it is due either to inclusion of the small z-data in our fit or to the different assumptions in the global fit parametrizations.

Acknowledgments

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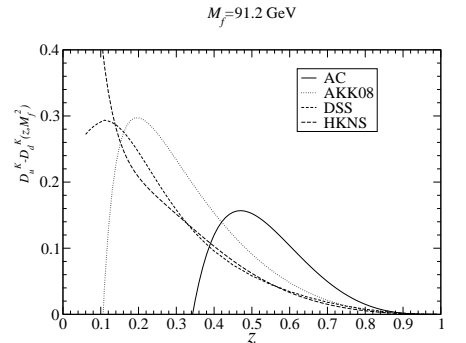
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Figure 1: $D_{u-d}^{K^+ + K^-}$ as obtained in this paper (labeled AC) and as obtained from the parametrizations of DSS, HKNS and AKK08.